

THIRD EDITION

AN INTRODUCTION TO NUMERICAL METHODS AND ANALYSIS



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Third Edition

James F. Epperson Mathematical Reviews



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PREFACE

I began working on this project in the late 1990s, and am very pleased that it has been successful enough to go through four editions¹ in slightly more than 20 years. My goal was ambitious: To write a text that was complete, yet could be used for an introductory course. Over the life of the text, the author has had to deal with changes in the computing environment available to students, as well as changes in the publishing/textbook industry. Because I have always seen this as an entry-level text, a lot of the new material, which might be considered non-elementary, is discussed largely through examples.

In this Third Edition, I have expanded the material on spectral methods, completely revamped the material on Poisson equations, included an additional method for the symmetric eigenvalue problem, added material on high-order difference methods for boundary value problems, and of course fixed (or tried to) several typo issues found and submitted to me by numerous colleagues. The dependence on MATLAB (or similar platforms) has increased a bit.

As with the Second Edition, there will be a website devoted to the text, maintained by the author. The URL should be something like:

www.jfepperson.org/3edition-web

If the "slings and arrows of outrageous fortune" should render that invalid, a link will certainly be posted on the Second edition website, which is here:

www.jfepperson.org/2edition-web.

The list of people to be thanked is lengthy, ranging from my original editor at Wiley, Barbara Holland, my current editor, Sandra Grayson (and her staff—most especially Kimberly Monroe-Hill, who put up with endless emails from me, and Becky Cowan, who led

¹The First Edition was published in 2001, the Revised Edition in 2007, the Second Edition in 2013, and this Third Edition in 2021.

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the effort that resulted in the outstanding cover for this edition), to my family—wife Georgia, children Elinor and Jay—as well as the staff and leadership at Mathematical Reviews (Executive Editor Edward Dunne) and the American Mathematical Society (Executive Director Catherine Roberts), who made possible the Study Leave (sabbatical) during which I wrote most of this new edition. A special shout-out is due to my MR Editorial colleague James Tian, who helped with a number of LaTeX issues (as did Amanda Francis and Mike Jones).

JFE

James F. Epperson

Ann Arbor

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